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International Journal of Heat and Mass Transfer 51 (2008) 3615–3622

International Journal of HEAT and MASS TRANSFER

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Numerical simulation of compression of the single spherical vapor bubble on a basis of the uniform model

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Received 30 August 2007; received in revised form 10 September 2007
Available online 31 December 2007

Abstract

The problem of the response of a single spherical vapor bubble is considered for the case of an abrupt increase of pressure in the surrounding infinite liquid. The mathematical model adopted is based on the assumption of the uniformity of pressure, temperature and density throughout the bubble volume. The temperature field around the bubble is calculated using the energy equation for the liquid. Thermal–physical characteristics, exclusive of specific heats of the liquid and vapor, are considered to be temperature-dependent. A notable feature of the model is the exact fulfillment of the integral law of conservation of system energy, disregarding the relatively small vapor kinetic energy. The initial bubble radius and the pressure rise in the liquid were varied in the calculations. It was found that the temperature increment in the bubble due to vapor condensation and heat exchange with the liquid is approximately two orders of magnitude less than that due to adiabatic compression. To study the effect of condensation, calculations were performed in which phase transitions were artificially blocked at the bubble boundary. It was found that the character of the process in the latter case changes both quantitatively and qualitatively; in particular, the temperature increment increases by about an order of magnitude.

Keywords: Steam bubble; Phase change; Heat-mass transfer; Sonoluminescence; Cavitation

1. Introduction

The prime object of the work is determination of a temperature increment in a vapor bubble and surrounding liquid by compression of the bubble due to the external pressure. As opposed to researches directed towards the study on bubbles dynamics, a correct consideration of heat exchange inside and outside the bubble and mass exchange at the interface is of fundamental importance for the problem of determination of temperature in the vapor or gas bubble. In the case of the vapor bubble, heat exchange in the liquid is most important among the above processes, as it affects temperature in the bubble not only directly, governing a heat inflow and outflow from the bubble, but also indirectly, controlling variation in vapor mass in the bubble because of phase transitions.

The consideration of heat exchange in the surrounding liquid makes the problem essentially intricate, since this generates a need for solving the energy equation for the liquid, that, as distinct from the rest of equations, is a partial differential equation. Papers where serious attention has been given to heat exchange in the liquid may be divisible into two groups. Among the first group are papers [1–5] in which approximate analytical approaches to solving the energy equation were used. An approximate method proposed in [6] was used most often. The method is based on an assumption that a thickness of a non-uniformly warmed-up layer of the liquid around the bubble is far less than the bubble size. It is apparent that such an approach is better suitable for problems of bubble

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doi:10.1016/j.ijheatmasstransfer.2007.10.014

Nomenclature

A	work done by external pressure forces
B	parameter in Eq. (30)
c	liquid specific heat capacity
c_V	heat capacity of the vapor at constant volume
c_p	specific heat capacity at constant pressure
K	kinetic energy of the liquid
E	internal energy
E_σ	surface energy
E_ψ	parameter in Eq. (30)
i	specific enthalpy
j	mass of vapor condensing per unit time on unit area of the interface
F	parameter in Eq. (27)
q	heat flux
p	pressure
r	radial coordinate
R	radius of the bubble
R_V	gas constant
S	area of the bubble surface
t	current time
T	temperature
u	velocity
U	liquid velocity at the bubble boundary
x	coordinate
V	volume of the bubble
w	vapor velocity at the bubble

Greek symbols

α	parameter in Eqs. (53–56)
β	parameter in Eq. (29)
γ	ratio of specific heats
ε	specific internal energies
λ	thermal conductivity coefficient
μ	dynamic coefficient of viscosity
θ	temperature increment
ρ	density
σ	coefficient of surface tension
τ	friction stress
Φ	dissipative function
ψ	heat of phase transition

Subscripts

s	steam–water interface
v	vapor
0	initial value
L	liquid

expansion than for those of bubble compression. The second group consists of papers in which the energy equation for the liquid around the bubble was solved numerically [7]. This approach is universal, but some difficulties of the computational character resulting from the specificity of the problem in hand may here arise.

The mathematical model used in the work may be considered being a modification of the uniform-bubble model [7]. The modification resides in taking account of the temperature dependence of thermal–physical properties and in a more comprehensive allowance for phase transitions. A distinguishing feature of the model proposed is the exact fulfillment of the integral law of conservation of total energy involving internal energy of the liquid and vapor, kinetic energy of the liquid and surface energy.

1. Setting of a problem and mathematical model

We shall deal with the process of the vapor bubble compression in infinite liquid at a sudden rise of pressure up to the constant value of p_∞ at an infinite distance from the bubble. At the initial time, the liquid and vapor in the bubble were in equilibrium with each other and had the temperature T_0. We accept the following main physical assumptions. The liquid is incompressible, viscous (Newtonian) and heat-conducting, the vapor in the bubble – inviscid, heat-conducting and subject to the Clapeyron equation. Viscosity, thermal conductivity and surface tension coefficient are temperature-dependent, heat capacities of the liquid and vapor are constant. At the bubble surface the temperature of the vapor equals that of the liquid, and the vapor pressure corresponds to the liquid temperature on the saturation curve.

The above physical assumptions (with the exception of temperature dependencies of viscosity and surface tension coefficient) are conventional for the class of problems at hand, however, their mathematical description can be realized in different ways. Below is proposed the mathematical model of the uniform-bubble based on the two following supplementary assumptions. Firstly, pressure and temperature (and, consequently, density) of the vapor are taken to be uniform through the volume of the bubble including interface. Secondary, it is suggested that the vapor velocity is negligible as compared to the liquid velocity at the interface. Then conditions at the phase boundary may be represented in the form [8]:

$$p_S - p_V = \tau_S - \frac{2\sigma}{R} - jU \quad (1)$$

$$q_S - q_{VS} = j\psi + \frac{j}{\rho}\frac{2\sigma}{R} - \frac{d\sigma}{dt} + \frac{1}{2}jU^2 \quad (2)$$

$$\psi = i_V - i_S^* \quad (3)$$

Hereafter the presence or absence of index "V" implies that the given parameter refers to vapor or liquid, respectively, the index "S" designates parameters at the bubble surface.

In Eqs. (1)–(3), p is the pressure, $\tau = \tau_{rr}$ is the friction stress, σ is the surface tension coefficient, R is the bubble radius, j is the specific mass flow of vapor condensing at the bubble surface, U is the liquid velocity at the bubble boundary, q_S is the specific heat flux from the bubble surface to the liquid, q_{VS} is the specific heat flux from the vapor to the bubble surface, ρ is the density, i is enthalpy, i_S^* is liquid enthalpy at vapor temperature and pressure at the interface. From Eqs. (2) and (3) it follows that ψ constitutes a heat of phase transition in the isobaric–isothermal conditions at a flat surface and at infinitely small velocities of the liquid and vapor. From this point on we will call this parameter as the thermodynamic heat of phase transition.

Specific internal energies and specific enthalpies of the liquid and vapor can be defined as follows:

$$\varepsilon = c\theta + \varepsilon_0, \quad \varepsilon_V = c_V T_V + \varepsilon_{V0}, \quad i = \varepsilon + \frac{p}{\rho},$$
$$i_V = \varepsilon_V + \frac{p_V}{\rho_V} = c_p T_V + \varepsilon_{V0} \tag{4}$$

Here $\theta = T - T_0$, c is the specific heat of the liquid,, c_V,c_p are specific heats of the vapor at constant volume and constant pressure. Constants ε_0 and ε_{V0} (note in the problem at hand one of the two constants may be taken arbitrarily) are linked between each other through the thermodynamic heat of phase transition. Designating the latter at the initial temperature T_0 and vapor pressure p_{V0} as ψ_0, one can derive from Eqs. (3) and (4):

$$\varepsilon_{V0} = \varepsilon_0 + \psi_0 - c_p T_0 + \frac{p_{V0}}{\rho} \tag{5}$$

$$\psi = \psi_0 + \theta_S(c_p - c) - \frac{(p_V - p_{V0})}{\rho} \tag{6}$$

For the Newtonian incompressible liquid in the case of spherical symmetry, the equation of motion, solution of the continuity equation and kinematic condition at the bubble surface may be written as follows:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial r} + \frac{\partial \tau}{\partial r} + \frac{2}{r}\left(\tau - \frac{2\mu u}{r}\right), \tau = 2\mu\frac{\partial u}{\partial r} \tag{7}$$

$$u = \frac{1}{r^2}UR^2 \tag{8}$$

$$\frac{dR}{dt} = U - \frac{j}{\rho} \tag{9}$$

Here t, r – time and radial coordinate, u – velocity, μ – dynamic viscosity, $\tau = \tau_{rr}$ – friction stress. From Eq. (8) it follows that the product r^2u is independent of radius. Integrating Eq. (7) with respect to radius from $r = R$ to $r = \infty$ and taking into account Eqs. (1), (8), (9), we obtain the following equation:

$$\rho\left(R\frac{dU}{dt} + \frac{3}{2}U^2\right) = p_V - p_\infty - \frac{2\sigma}{R} - \Omega + jU,$$
$$\Omega = 12UR^2\int_R^\infty \mu r^{-4}dr \tag{10}$$

We will write the energy equation for the liquid surrounding the bubble:

$$\rho c\left(\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\lambda\frac{\partial \theta}{\partial r}\right) + \Phi, \quad \Phi = 12\mu U^2R^4r^{-6} \tag{11}$$

Here, λ is the thermal conductivity, Φ is the dissipative function. We introduce a new coordinate, $x = r - R(t)$, related to the moving boundary of the bubble. In the new coordinate system Eq. (11), in terms of Eq. (8), will take the form

$$\rho c\left(\frac{\partial}{\partial t}((R+x)^2\theta)\right)_x + \rho c\frac{\partial}{\partial x}\left(\left(UR^2 - (R+x)^2\frac{dR}{dt}\right)\theta\right)$$
$$= \frac{\partial}{\partial x}\left((R+x)^2\lambda\frac{\partial \theta}{\partial x}\right) + \frac{12\mu U^2R^4}{(R+x)^4} \tag{12}$$

The initial and boundary conditions for Eq. (12) are as follows:

$$\theta|_{t=0} = 0, \quad \theta|_{x=\infty} = 0, \quad \theta|_{x=0} = \theta_S(t) = T_S - T_0 \tag{13}$$

Consider now processes within the bubble. In the general case the Clapeyron, energy and continuity equations for the inviscid ideal gas have the form

$$p_V = \rho_V R_V T_V \tag{14}$$

$$\rho_V c_p\left(\frac{\partial T_V}{\partial t} + w^i\nabla_i T_V\right) = \frac{\partial p_V}{\partial t} + w^i\nabla_i p_V - \nabla_i q_V^i \tag{15}$$

$$\frac{\partial \rho_V}{\partial t} + \nabla_i(\rho_V w^i) = 0 \tag{16}$$

Here R_V is the vapor gas constant, w^i, q_V^i are components of velocity and heat flux in the vapor. Using Eqs. (14), (16) and introducing adiabatic exponent $\gamma = c_p/c_V$, we can transform Eq. (15) into the form

$$\frac{1}{\gamma - 1}\frac{\partial p_V}{\partial t} + c_p\nabla_i(\rho_V w^i T_V) = w^i\nabla_i p_V - \nabla_i q_V^i \tag{17}$$

Integrating Eq. (17) with respect to volume and using the Gauss theorem, we will obtain:

$$\frac{1}{\gamma - 1}\int_V \frac{\partial p_V}{\partial t}dV + c_p\int_S \rho_V w^i T_V n_i \, dS$$
$$= \int_V w^i\nabla_i p_V dV - \int_S q_V^i n_i \, dS \tag{18}$$

Here n_i are components of the normal to the surface S limiting the vapor volume V. For the spherically symmetric problem Eq. (18) takes the form

$$\frac{1}{\gamma - 1}\int_V \frac{\partial p_V}{\partial t}dV + c_p\rho_{VS}w_S T_{VS}S$$
$$= \int_V w\frac{\partial p_V}{\partial r}dV - Sq_{VS} \tag{19}$$

Herew_S, ρ_{VS}, and T_{VS} are the velocity, density and temperature of the vapor at the bubble surface. From Eq. (16) it follows that

$$\rho_{VS}w_S = \rho_{VS}\frac{dR}{dt} + j \tag{20}$$

Substituting Eq. (20) in Eq. (19) and making some transformations, we will obtain the energy equation of the vapor in the form

$$\frac{1}{\gamma-1}\int_V \frac{\partial p_V}{\partial t}dV + \frac{\gamma}{\gamma-1}p_{VS}\frac{dV}{dt} + c_pJT_{VS} = \int_V w\frac{\partial p_V}{\partial r}dV - Sq_{VS} \tag{21}$$

Here, p_{VS} is the vapor pressure at the bubble boundary, $J = jS$. Allowing for that in the context of the model at hand, pressure and temperature are uniform through the bubble volume and using Eq. (2), we will present the integral energy equation for the vapor (21) in the form

$$\frac{1}{\gamma-1}\frac{d(p_VV)}{dt} + p_V\frac{dV}{dt} = -Sq_S + J\psi + \frac{J}{\rho}\frac{2\sigma}{R} - S\frac{d\sigma}{dt} + \frac{1}{2}JU^2 - c_pJT_V, \quad q_S = -\lambda\frac{\partial\theta}{\partial x}\Big|_{x=0} \tag{22}$$

Within the limits of taken assumptions, the equation of mass balance for the vapor in the bubble may be written as follows:

$$\frac{dm_V}{dt} = \frac{d}{dt}(\rho_VV) = -J \tag{23}$$

For closing of the problem, it is required to use the saturation curve in some form, for example, in the form of the modified Clausius–Clapeyron equation:

$$\frac{dT_V}{dt}\frac{\rho_V}{T_V}\left(\frac{\psi}{R_VT_V} - 1\right) = \frac{d\rho_V}{dt} \tag{24}$$

Eqs. (6), (9), (10), (12), (14), (22), (23), (24) form a closed system of eight equations for eight main unknowns: $U,R,\rho_V,T_V,p_V,\theta,\ \psi,j$.

Denote kinetic and internal energy of the liquid by K and E, vapor internal energy – by E_V, the surface energy – by E_σ, the work done by external pressure – by A:

$$K = 2\pi\rho U^2R^3, \quad E = 4\pi\int_0^\infty \rho\varepsilon(R+x)^2dx,$$
$$E_V = m_V\varepsilon_V, \quad E_\sigma = \sigma S, \frac{dA}{dt} = -USp_\infty \tag{25}$$

Then, from equations constituting the above mathematical model one can derive using identical mathematical transformations, the following expression:

$$\frac{dK}{dt} + \frac{dE}{9hdt} + \frac{dE_V}{dt} + \frac{dE_\sigma}{dt} = \frac{dA}{dt} - \rho US\varepsilon_0 \tag{26}$$

For the problem at hand, Eq. (26) is an exact expression of the energy conservation law without regard for the vapor kinetic energy (in the above model proposed, the vapor velocity is considered being negligible). Integrating Eq. (26) in view of the initial conditions, after some transformations, another form of the energy conservation law can be obtained:

$$F = \sigma_0S_0 + \frac{p_{V0}V_0}{\gamma-1} + \beta m_{V0} + p_\infty V_0 \tag{27}$$

$$F = K + 4\pi\rho c\int_0^\infty \theta(R+x)^2dx + \sigma S + \frac{p_VV}{\gamma-1} + \beta m_V + p_\infty V \tag{28}$$

$$\beta = \left(\psi_0 - c_pT_0 + \frac{p_{V0}-p_\infty}{\rho}\right) \tag{29}$$

Here, m_{V0} is the initial vapor mass in the bubble. Note the right-hand side of Eq. (27) is independent of time. This implies the magnitude F is also independent of time, i.e. it is a conservation integral in the context of the model of the process at hand. For the further analysis it is convenient to transform Eq. (27) as follows:

$$K + B_L + B_V = \Delta A + \Delta E_\sigma + \Delta E_\psi \tag{30}$$

$$B_L = 4\pi\rho c\int_0^\infty \theta(R+x)^2dx, \quad B_V = c_Vm_V(T_V - T_0),$$
$$\Delta A = p_\infty\left(V_0 - V - \frac{m_{V0}-m_V}{\rho}\right), \tag{31}$$
$$\Delta E_\sigma = \sigma_0S_0 - \sigma S, \quad \Delta E_\psi = (m_{V0} - m_V)(\varepsilon_V - \varepsilon)|_{T=T_0}$$

2. Fundamental constituents of the computational algorithm and results of calculations

For numerical realization of the above mathematical model, the difference scheme, having, as a whole, the first order of accuracy in time, was used. Since some different equations contained more than one unknown, iterations of the specific mass flux were conducted at each time step to find a solution. An accuracy of the fulfillment of the difference analog of Eq. (22) served as a criterion for completion of iterations. The energy Eq. (12) for the liquid was approximated by the implicit difference scheme and solved by the marching method. In doing so, meshes nonuniform in space with a crowding near bubble boundaries were used.

With using preliminary methodical experiments, optimal mesh parameters were found for each case. As the rate of processes at hand increases essentially with time, a variable step in time, changed in proportion to the current size of the bubble, was used in calculations. Initial values of time steps and minimum steps of spatial mesh were taken as follows: $\Delta t = 10^{-7}$ s, $\Delta x = R_0/32000$ at $R_0 = 10^{-3}$ m; $\Delta t = 10^{-9}$ s, $\Delta x = R_0/16000$ at $R_0 = 10^{-4}$ m; $\Delta t = 10^{-11}$ s, $\Delta x = R_0/8000$ at $R_0 = 10^{-5}$ m.

It is necessary to note the used computational algorithm as a whole has no conservatism property, therefore, the difference analog of Eq. (30) was not exactly fulfilled in numerical calculations. In the process of researches it has been ascertained that a designed value of the residual of the integral energy balance is an extra convenient tool for control of miscalculations associated, e.g., with choosing meshes in space and time. The results below were obtained by calculations where the residual mentioned did not exceed a few percent.

In the mathematical model used it is anticipated that the vapor velocity is negligible when compared with the liquid velocity. To check this assumption, the vapor velocity at the interface was calculated by the formula

$$w_{\mathrm{S}} = \frac{\mathrm{d}R}{\mathrm{d}t} + \frac{j}{\rho_{\mathrm{V}}} \tag{32}$$

For the majority of calculations conducted the ratio w_{S}/U did not exceed 10%.

For the approval of the calculation technique were used experiments [4]. In these experiments, the dynamics of the vapor bubble in water was investigated into with an increase in the external pressure from the initial value p_{V0} up to the atmospheric one p_{a}. The dynamics of time variation of pressure in the course of experiments was recorded, and the data were used in calculations. Results of the comparison are given in Fig. 1.

Now we will go to the results of numerical simulation according to the above model. In all the calculations, evolution of the vapor bubble in water was considered. Temperature dependencies of thermal and physical properties of water were interpolated by data [9]. In all the calculations the initial temperature was taken as 293 K, the initial pressure in the bubble was equal to the saturation vapor pressure at this temperature (2337 Pa). Pressure in the surrounding liquid increased step-wise. In the calculations, the bubble initial radius and pressure jump in the liquid were varied. In all the cases the calculations were ceased in the cases when the liquid velocity at the bubble boundary had attained the sonic speed.

Shown in Fig. 2 is time variation of the radius of bubbles. The curves 1–3 correspond to different initial radii of bubbles at an identical pressure jump in the liquid, the curves 2, 4, 5 – to different pressure jumps at identical initial radii. The bubble boundary velocity obtained in the calculations was compared with a velocity calculated by the Rayleigh formula [10]:

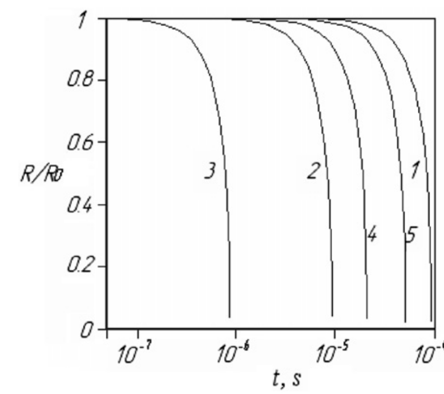

Fig. 2. Time variation of the radius of bubbles. curve 1: $R_0 = 10^{-3}$ m, $P_\infty = 10^5$ Pa; curve 2: $R_0 = 10^{-4}$ m, $P_\infty = 10^5$ Pa; curve 3: $R_0 = 10^{-5}$ m, $P_\infty = 10^5$ Pa; curve 4: $R_0 = 10^{-4}$ m, $P_\infty = 2 \times 10^4$ Pa; curve 5: $R_0 = 10^{-4}$ m, $P_\infty = 4 \times 10^3$ Pa.

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \sqrt{\frac{2}{3}\frac{p_\infty - p_{V0}}{\rho}\left(\left(\frac{R_0}{R}\right)^3 - 1\right)} \tag{33}$$

The Rayleigh formula is an exact solution of the problem in hand provided that the pressure in the bubble is constant and the surface tension and viscosity of the liquid are not taken into account. In the calculation to which curve 5 corresponds the distinction in velocities comprised about 20%, in the rest of cases-essentially less (a few percent).

Shown in Fig. 3 is the temperature increment in the bubble as a function of the compression ratio (R_0/R) at different initial conditions. As is seen, the greater is the initial radius of the bubble and the higher is the pressure jump in the liquid, the higher the temperature increment at the same compression ratio. With the bubble compression by a factor of about thirty the maximum temperature

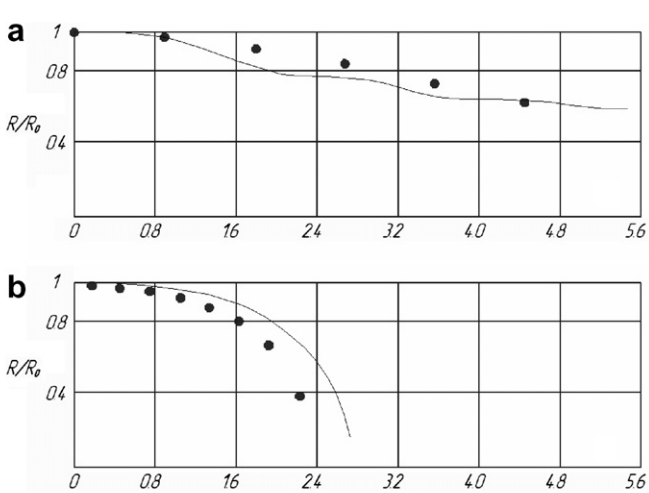

Fig. 1. Comparison of calculated results with experimental data (calculation–full lines, experiment-circles, $t_* = \frac{t}{R_0}\sqrt{\frac{2}{3}\frac{(p_{\mathrm{a}} - p_{\mathrm{V0}})}{\rho}}$): for the cases: (a) – $R_0 = 4.64$ mm, $p_{\mathrm{V0}} = 58562$ Pa, $T_0 = 85$ °C; (b) –$R_0 = 8.89$ mm, $p_{\mathrm{V0}} = 15756$ Pa, $T_0 = 55$ °C.

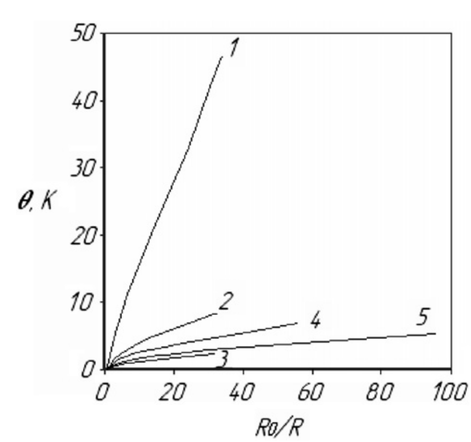

Fig. 3. Temperature within the vapor bubble curve 1: $R_0 = 10^{-3}$ m, $P_\infty = 10^5$ Pa; curve 2: $R_0 = 10^{-4}$ m, $P_\infty = 10^5$ Pa; curve 3: $R_0 = 10^{-5}$ m, $P_\infty = 10^5$ Pa; curve 4: $R_0 = 10^{-4}$ m, $P_\infty = 2 \times 10^4$ Pa; curve 5: $R_0 = 10^{-4}$ m, $P_\infty = 4 \times 10^3$ Pa.

increment in the calculations conducted made up approximately 50 K. It is interesting to compare this quantity with the temperature increment at the adiabatic compression of the bubble (i.e. with the complete absence of heat and mass exchange between the vapor in the bubble and surrounding liquid). It is easy to show that with the adiabatic compression of the bubble by a factor of thirty times and at the initial temperature of 290 K, temperature must rise approximately up to 8500 K. Such a great (by two orders) distinction in the temperature increment gives an indication of the controlling role of heat and mass exchange processes in the problem at hand.

Heat and mass exchange parameters (specific heat flux q_S and specific mass flux j) as a function of the compression ratio are presented in Fig. 4. We point out that the form and relative position of pertinent curves on both plots are similar to each other, i.e. there is a correlation between heat and mass fluxes.

The availability of this correlation can be seen from analysis of obtained results in the view of the conservation integral in the form of Eq. (30). The components in the left-hand side of Eq. (30) constitute increments (relative to the initial state) of different kinds of energy: kinetic energy of the liquid K, internal energy of the liquid B_L and internal energy of the vapor remaining in the bubble B_V. Accordingly, the components in the right-hand side of Eq. (31) constitute sources of these increments: the work done by the external pressure ΔA, released surface energy ΔE_σ and latent condensation energy ΔE_ψ. Values of these quantities (except for B_V that is much less than the rest) for one of the calculations are shown in Fig. 5. It is seen the main source of a liquid warm-up is the condensation energy whereas the work done by the external pressure goes almost entirely to increase the liquid kinetic energy. Note as well, the released condensation energy exceeds perceptibly the work done on the system.

Results presented reveal the condensation is one of the determining processes for the problem at hand. To reveal more fully effects associated with the condensation, calculations where the phase transition was artificially blocked up have been performed. For this purpose it was believed in all equations that $j = 0$, the saturation curve equation was excluded from the mathematical model. This problem is completely equivalent to that of the gas bubble compression in the absence of dissolution, i.e. at a constant gas mass in the bubble, provided that gas properties are coincident with the vapor ones. And so, for convenience sake, from here on the vapor in the problem with the "blocked up" phase transition will be called gas.

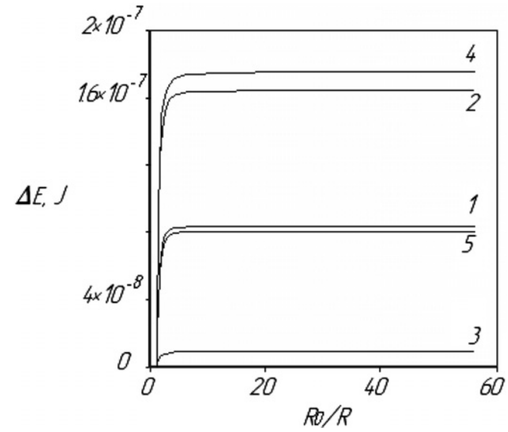

Fig. 5. Constituents of the conservation integral (30) for the case $R_0 = 10^{-4}$ m, $P_\infty = 2 \times 10^4$ Pa (curve 1 – ΔA, curve 2– ΔE_ψ, curve 3 – ΔE_σ, curve 4 – B_L, curve 5 – K).

One of radical distinctions in evolution of vapor and gas bubbles at an increase of the external pressure consists in that the vapor bubble can monotonically contract up to a full disappearance (collapse regime), whereas in the case of the gas bubble, the initial compression necessarily changes to expansion, once a certain minimum size has been achieved.

For conditions in which the comparison of evolutions of the vapor and gas bubbles was carried out ($R_0 = 10^{-4}$ m, $P_\infty = 2 \times 10^4$ Pa), this minimum radius of the gas bubble corresponds to the compression ratio $R_0/R \approx 19$ (Figs. 6,

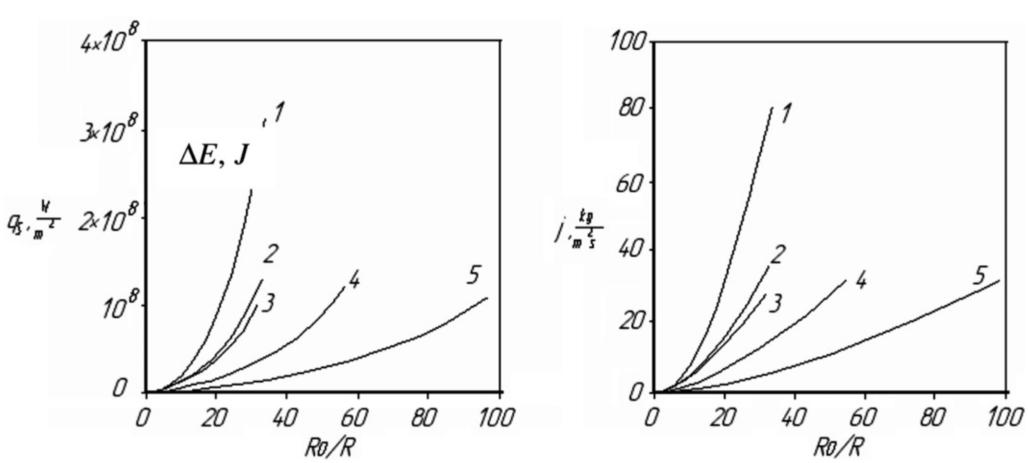

Fig. 4. Specific heat flux from the interface to the liquid and specific mass flux of condensing vapor. curve 1: $R_0 = 10^{-3}$ m, $P_\infty = 10^5$ Pa; curve 2: $R_0 = 10^{-4}$ m, $P_\infty = 10^5$ Pa; curve 3: $R_0 = 10^{-5}$ m, $P_\infty = 10^5$ Pa; curve 4: $R_0 = 10^{-4}$ m, $P_\infty = 2 \times 10^4$ Pa; curve 5: $R_0 = 10^{-4}$ m, $P_\infty = 4 \times 10^3$ Pa.

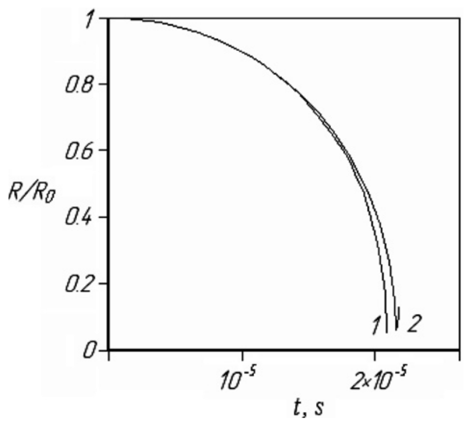

Fig. 6. Variation in the radius of the vapor (1) and gas (2) bubbles.

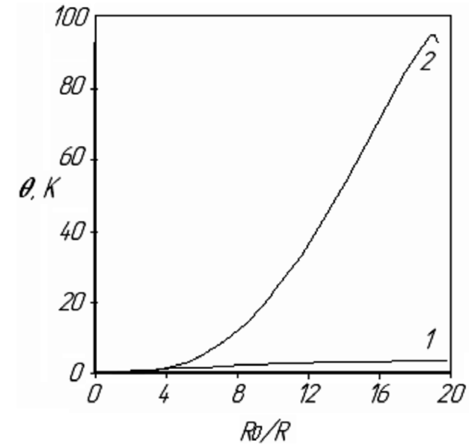

Fig. 8. Variation in temperature for the vapor (1) and gas (2) bubbles.

and 7). Since in this paper the expansion regime is no subject of investigation, the comparative analysis of characteristics of vapor and gas bubbles will be limited by the compression ratio mentioned. Note the vapor bubble compression rate grows steadily with a decrease in the vapor radius (Fig. 7). This demonstrates a greater probability of realization of the vapor bubble collapse regime in the conditions at hand.

Shown in Figs. 8, and 9 are temperature increments and heat fluxes to the liquid for the vapor and gas bubbles. At the initial stage of the process, temperature in the vapor bubble is somewhat higher than in the gas bubble. However, at a later time at the identical compression ratio, the temperature increment and heat flux to the liquid are much higher in the case of the gas bubble. The same applies to a growth of the pressure and density – at the maximum compression ratio the pressure and density in the gas bubble exceed approximately the initial values by 9000 and 7000 times, whereas for the vapor bubble the excess comprises about 50%. When analyzing results presented it is necessary to consider that the identical compression ratio of bubbles is appropriate to different instants of time.

Given in Fig. 10 are values of different components in the conservation integral (30) for the gas bubble. As differentiated from the case of the vapor bubble, at the compression ratio appropriate for the minimum radius, kinetic energy of the liquid turns to zero, going to thermal energy of liquid and gas. In doing so, like for the vapor bubble, the thermal energy increment of the liquid exceeds essentially that of the vapor. And so, the work done by external forces at the maximum compression ratio is equal, for all

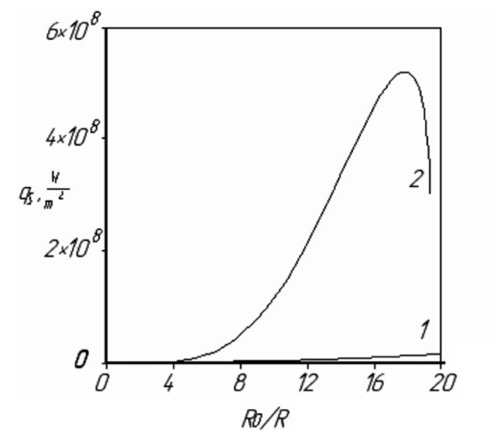

Fig. 9. Variation in the heat flux for the vapor (1) and gas (2) bubbles.

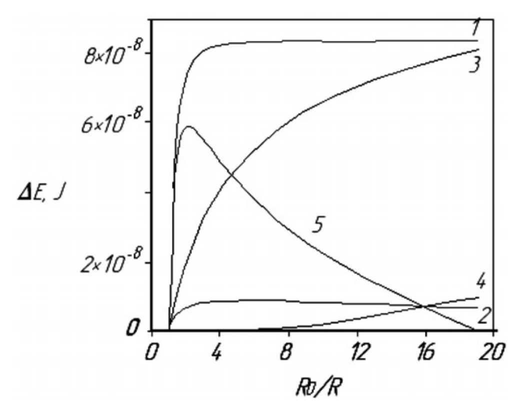

Fig. 10. Constituents of the conservation integral (30) for the gas bubble ($R_0 = 10^{-4}$ m, $P_\infty = 2 \times 10^4$ Pa). 1 – ΔA, 2 – ΔE_σ, 3 – B_L, 4 – B_V, 5 – K.

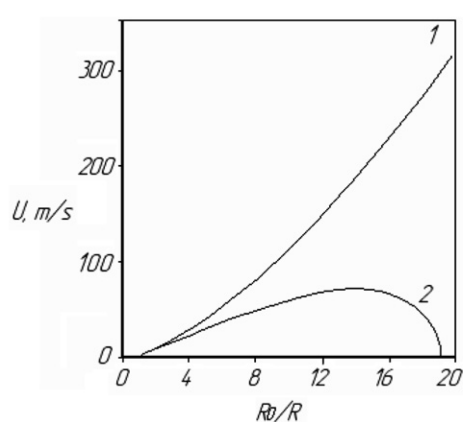

Fig. 7. Liquid velocity at the bubble boundary: 1 – vapor bubble, 2 – gas bubble.

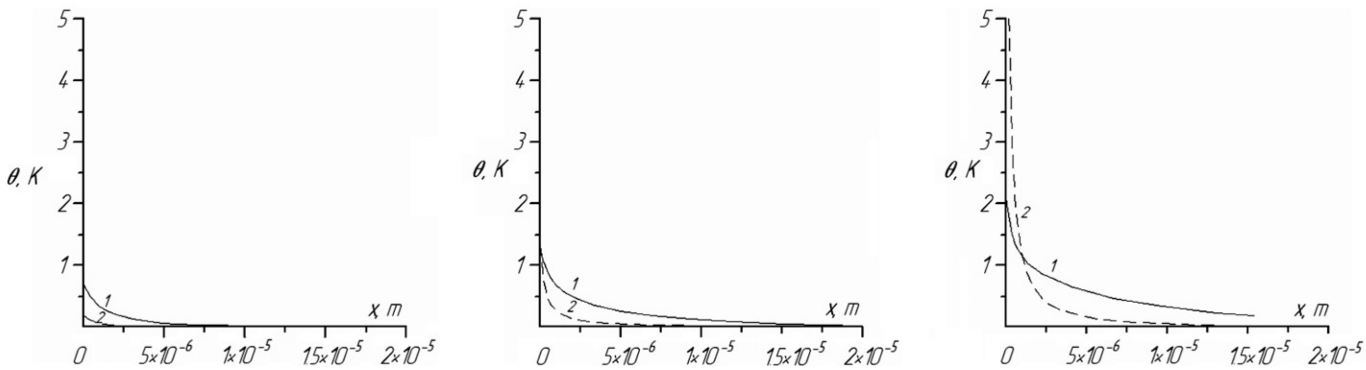

Fig. 11. Temperature profiles in the liquid for the vapor (1) and gas (2) bubbles (a – $R_0/R = 2$, b – $R_0/R = 4$, c – $R_0/R = 8$).

practical purposes, to the thermal energy increment of the liquid. Note that in spite of an addition due to kinetic energy, the quantity of the thermal energy increment of the liquid for gas is perceptibly less than for vapor, though the liquid surface temperature in the case of gas is much higher. This is because the liquid around the gas bubble is warmed-up through the lesser depth than around the vapor bubble (Fig. 11). This effect is likely to result from a different role of convective heat transfer in the liquid, as the liquid velocity in the case of the vapor bubble is much higher (Fig. 7).

2. Conclusions

The mathematical model for the uniform vapor bubble, based on assumptions of the uniformity of pressure and temperature through the bubble volume and of a negligible vapor velocity at the bubble surface (when compared to the liquid velocity), has been proposed. As opposed to other models, in the model proposed, the temperature dependence of thermal and physical properties (exclusive of specific heats of liquid and vapor) is taken into account, as well as effects caused by phase transitions are more fully allowed for. The temperature field around the bubble is derived from the energy equation for the liquid. The model ensures the exact fulfillment of the integral law of conservation of total energy including kinetic and internal energies of the liquid, internal energy of the vapor and surface energy. In the context of the model proposed the conservation integral for the case of stepwise change of pressure in the liquid has been found.

On a basis of numerical realization of the proposed model, theoretical research of processes taking place in the vapor bubble and surrounding liquid (water) with a stepwise increase in pressure at an infinite distance from the bubble has been carried out. It has been obtained, the vapor in the bubble contracts essentially less than the bubble itself due to the condensation at the interface. The work done thereat on the system by the external pressure changes practically fully to kinetic energy of the liquid, and the liquid velocity is well described by the Rayleigh formula conjecturing that the pressure in the bubble is constant. The thermal energy increment of the liquid exceeds vastly that of the vapor which points up to an intense heat removal to the liquid. The increment of the liquid internal energy occurs, for the most part, due to an energy released through the vapor condensation at the bubble boundary.

The effect of condensation on processes was studied through artificial "blocking up" of the phase transition. The problem in the present formulation is equivalent to the case of the bubble filled in by insoluble gas whose properties are coincident with those of water vapor. It has been obtained in the absence of the condensation, at an instant of time of changing from compression of the bubble to its expansion, a temperature increment within the bubble exceeds that with the available condensation at the same compression ratio by more than an order. For this instant of time, the work done on the system by the external pressure coincides practically with the liquid internal energy increment. It has been established that at the identical compression ratio, the liquid around the bubble has been warmed-up to a greater depth with the available condensation that in the absence thereof.

References

[1] M.S. Plesset, S.A. Zwick, The growth of vapor bubbles in superheated liquids, J. Appl. Phys. 25 (4) (1954) 493–500.
[2] H.K. Forster, N. Zuber, Growth of a vapor bubble in a superheated liquid, J. Appl. Phys. 25 (4) (1954) 474–478.
[3] S.A. Zwick, M.S. Plesset, On the dynamics of small vapor bubbles in liquids, J. Math. Phys. 33 (4) (1955) 308–330.
[4] L.W. Florschuetz, B.T. Chao, On the mechanism of vapor bubble collapse, Trans. ASME 87 (2) (1965) 58–72.
[5] S.M. Cho, R.A. Seban, On some aspects of steam bubble collapse, Trans. ASME 91 (4) (1969) 104–111.
[6] M.S. Plesset, S.A. Zwick, A nonsteady heat diffusion problem with spherical symmetry, J. Appl. Phys. 23 (1952) 95–98.
[7] R.I. Nigmatulin, Dynamics of Multiphase Media V. 1, Hemisphere, 1990.
[8] L.I. Sedov, Mechanics of Continuous Media, World Scientific Publishing Co. Inc., 1996.
[9] N.B. Vargaftik, Handbook of Physical Properties of Liquids and Gases: Pure Substances and Mixtures, Bagell House Publishers, 1996.
[10] Lord Rayleigh, On the pressure developed in a liquid during the collapse of a spherical cavity, Philos. Mag. 34 (1917) 94–98.